Advanced Topics

UNIT 6

Probability

Advanced Topics in Mathematics Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 6.1 Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Predicting Outcomes

**Fundamental Counting Principle:**

If event M can occur in m ways and event N can occur in n ways, then event M followed by event N can occur in m · n ways.

**EXAMPLE 1:** Calculate the possible outcomes of each event.

|  |  |  |
| --- | --- | --- |
| 1. Suppose that the graduation requirement for some college was that a student take one course in mathematics and one course in computer science. If there were 5 math courses and 4 computer science courses available, in how many different ways could a student meet this requirement?
 | 1. How many telephone numbers are possible for the area code 859? Assume that there are no restrictions on the remaining digits in the phone number.
 | 1. If an objective test consists of 10 true/false questions and 10 multiple-choice questions (each with four options), in how many different ways could the answer sheet be filled out?
 |

**EXAMPLE 2:** The latest password requirements on most digital applications requires 8 characters, at least one uppercase, one lower case, and one symbol from the list {@, #, $, %, \*, <, or >}. How many possible 8-character combinations are there?

**EXAMPLE 3:** There are twelve members on Yen’s baseball team. How many different ways can a batting order be made of nine players?

**EXAMPLE 4:** Consider the word *numbers*.

|  |  |
| --- | --- |
| 1. How many different arrangements are there of the letters of the word *numbers*?
 | 1. How many arrangements begin with a vowel?
 |

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Predicting Outcomes

 **PRACTICE SET A**

1. A restaurant offers a choice of 4 salads, 5 main courses, and 4 desserts. How many possible 3-course meals are there?
2. You are taking a multiple-choice test that has 7 questions. Each of the questions has 5 choices, with one correct choice per question. If you select one of these options per question and leave nothing blank, in how many ways can you answer the questions?
3. There are 7 performers who are to present their acts at a variety show. One of them insists on being the first act of the evening. If this request is granted, how many different ways are there to schedule the appearances?
4. In a contest in which 8 contestants are entered, in how many ways can the 5 distinct prizes be awarded?
5. An ice chest contains 6 cans of apple juice, 8 cans of grape juice, 5 cans of orange juice, and 4 cans of pineapple juice. Suppose that you reach into the container and randomly select three cans in succession. How many ways can you select anything EXCEPT grape juice?
6. **Powerball**® is a combined large jackpot game and a cash game. Every Wednesday and Saturday night at 10:59 p.m. Eastern Time, we draw five white balls out of a drum with 59 balls and one red ball out of a drum with 35 red balls. How many possible lottery combinations exist?
7. **You are required to make a 4 digit PIN (Personal Identification Number). How any PIN’s can you form with the given restrictions:**
	1. **All odd numbers.**
	2. **No repeating digits.**
	3. **The number is divisible by 5.**
8. You are coaching an amateur softball team consisting of 15men and 8 women. It is a slow pitch, recreational league, so that we have 10 spots on a batting order (you got an extra “rover”, a player in between infielders and outfielders). How many ways to construct a batting order if….
	1. Women don’t bat.
	2. The leadoff (1st batter) must be a woman.
	3. The cleanup (4th batter) must be a man.
	4. 1st thru 4th are men and 5th thru 7th are women.
	5. Men and women bat alternatively (i.e. a man must bat after a woman, and vice versa)

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Probability and Odds

The probability of an event *E* occurring can be found with the formula:

$$P\left(E\right)=\frac{number of favorable outcomes}{total number of possible outcomes}$$

**EXAMPLE 1:** Find the probability given you are rolling a 12-sided die.

|  |  |  |
| --- | --- | --- |
| 1. P(4) means the probability of the outcome being a 4
 | 1. P(odd)
 | 1. P(greater than 5)
 |

**EXAMPLE 2:**  Given the Wheel of Fortune spinner to the right,
 determine the following probabilities:

1. P($100)
2. P($500)
3. P(no money)
4. P(at most $300)

**PRACTICE SET A**

Find each probability if a 6-sided die is rolled once.

1. P(2)
2. P(5)
3. P(even number)
4. P(odd number)
5. P(numbers less than 5)
6. P(numbers greater than 1)

Find each probability if a card is drawn from a standard deck of playing cards.

1. P(heart)
2. P(5 of hearts)
3. P(Ace)
4. P(black Ace)
5. P(face card)

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Probability and Odds

**OR Probabilities**

It is important to determine if the events are **Mutually Exclusive** or **Inclusive**.

**Mutually Exclusive** means the two events have no common occurrences.
$$P\left(A or B\right)=P\left(A\right)+P(B)$$

**Inclusive**means the two events have at least one common occurrence
$$P\left(A or B\right)=P\left(A\right)+P\left(B\right)-P(A and B)$$

**EXAMPLE 3:** Determine the probability for rolling a ten-sided die.

|  |  |  |
| --- | --- | --- |
| 1. P(1 or 10)
 | 1. P(at least 7)
 | 1. P(multiple of 2 or 3)
 |

**AND Probabilities**

Known as a compound probability where you are combining multiple events,
multiply the probability of each distinct event.
$$P\left(A and B\right)=P\left(A\right)∙P(B)$$

**EXAMPLE 4:** Two marbles are drawn with replacing from a bag containing 3 red, 5 blue, and 6 green marbles.

|  |  |  |  |
| --- | --- | --- | --- |
| 1. P(both red)
 | 1. P(blue, then green)
 | 1. P(at least one blue)
 | 1. P(both the same)
 |

**PRACTICE SET B**

**Find each probability if a 6-sided die is rolled twice.**

|  |  |
| --- | --- |
| 1. P(2, then 6)
2. P(two even #’s)
3. P(prime, then 5)
 | 1. P(two odds or two 3’s)
2. P(two of the same number)
3. P(two different numbers)
 |

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Probability and Odds

 **You choose a marble at random from a bag containing 3 blue marbles, 5 red marbles, and 2 green marbles. You replace the marble and then choose again. Find each probability.**

|  |  |
| --- | --- |
| 1. *P*(both blue)
2. *P*(both red)
3. *P*(blue then green)
 | 1. *P*(red then blue)
2. *P*(green then red)
3. *P*(both green)
 |

 **You choose a tile at random from a bag containing 2 tiles with X, 6 tiles with Y, and 4 tiles with Z. You pick a second tile without replacing the first. Find each probability.**

|  |  |
| --- | --- |
| 1. *P*(X then Y)
2. *P*(both Y)
3. *P*(Y then X)
 | 1. *P*(Z then X)
2. *P*(both Z)
3. *P*(Y then Z)
 |

1. There are 12 girls and 14 boys in math class. The teacher puts the names of the students in a hat and randomly picks one name. Then the teacher picks another name without replacing the first. What is the probability that both students picked are boys?
2. The probability that Bob will make a free throw is $\frac{2}{5}$. What is the probability that Bob will make
 his next two free throws?

**Find the odds of an event occurring given the probability of the event.**

|  |  |  |  |
| --- | --- | --- | --- |
| 1. $\frac{1}{2}$
 | 1. $\frac{11}{12}$
 | 1. $\frac{4}{7}$
 | 1. $\frac{4}{11}$
 |

**Find the probability of an event occurring, given the odds of the event.**

|  |  |  |  |
| --- | --- | --- | --- |
| 1. 6 : 1
 | 1. 5 : 6
 | 1. 9 : 8
 | 1. 7 : 9
 |

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Expected Values

***PLAY THE GAME ROULETTE!***

 We will play the game ten times. You can place any type of bet you wish. Everyone starts with 10 “chips,” or objects to place on the table. Keep track of your bets below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| BET | Your Bet(# chips : where) | What hit | Your Payback | TOTAL (10 chips starting) |
| 1 | : |  |  |  |
| 2 | : |  |  |  |
| 3 | : |  |  |  |
| 4 | : |  |  |  |
| 5 | : |  |  |  |
| 6 | : |  |  |  |
| 7 | : |  |  |  |
| 8 | : |  |  |  |
| 9 | : |  |  |  |
| 10 | : |  |  |  |

***The Odds***

 What are the odds of winning in a *straight-up* bet (successes to failures)? \_\_\_\_\_\_\_ : \_\_\_\_\_\_\_\_

 What is the pay-out (See table)? ­\_\_\_\_\_\_\_ : \_\_\_\_\_\_\_\_

Expected overall payout is determined by: [P(winning) x amount won] + [P(losing) x amount lost]

 Determine the expected payout for a straight-up bet.

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Expected Values

***What does this mean?***

 The “house” or casino should always make money off of you in a straight-up bet. What about other bets? Test two of your favorite bets from above to see.

1. What are the odds of winning in a \_\_\_\_\_\_\_\_\_\_\_\_\_ bet? \_\_\_\_\_\_\_ : \_\_\_\_\_\_\_\_

 What is the pay out? ­\_\_\_\_\_\_\_ : \_\_\_\_\_\_\_\_

 Determine the expected payout for a straight-up bet.

1. What are the odds of winning in a \_\_\_\_\_\_\_\_\_\_\_\_\_ bet? \_\_\_\_\_\_\_ : \_\_\_\_\_\_\_\_

 What is the pay out? ­\_\_\_\_\_\_\_ : \_\_\_\_\_\_\_\_

 Determine the expected payout for a straight-up bet.

***What do you think?***

 Is this game fair? What is the best bet for you? What is the worst bet for you? Write a paragraph below explaining.

***Homework***: Explore the gaming data for local casinos (<http://dunes.cincinnati.com/data/gaming/>). How much money did casinos make off Roulette in June? How much was made the entire year last year? What were the payouts? Write a one-page summary describing what you have learned about casino gaming and the risks to the player.

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Experimental v. Theoretical Probabilities

Monty’s Dilemma: Stick or Switch?

**PART A: Experiment**

|  |  |
| --- | --- |
| Stick | Switch |
| **WIN** | **LOSE** | **WIN** | **LOSE** |
|  |  |  |  |

**PART B: Computer Simulation**

|  |  |
| --- | --- |
| Stick | Switch |
| **# Trials** | **#Wins** | **#Trials** | **#Wins** |
|  |  |  |  |
| **Win Percentage** |  | **Win Percentage** |  |

**PART C: Theoretical Probability**

|  |  |
| --- | --- |
| Stick | Switch |

What do you think? Can you apply what you’ve learned to the updated version, Deal Or No Deal?

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Permutations and Combinations

**FACTORIAL**
Noted as an exclamation point, $n!=\left(n\right)\left(n-1\right)\left(n-2\right)…(2)(1)$
*Example:* $6!=6∙5∙4∙3∙2∙1=720$

When determining the possible outcomes of a compound event, we must be careful that each outcome is distinguishable. Notice the difference in the following situations:

* There are ten digits, 0, 1, 2, … 9 and you must select three for a password
* Ten books are labelled 0, 1, 2, … 9 and you are selecting three books to borrow

In the first, we determine the outcomes of each position of the password, obtaining the answer $10∙9∙8=720$. We know that the password **398** is different from the password **983**. Each password is unique and distinguishable, making what we call a ***permutation***.

In the second, though it seems there should also be 720 outcomes (ten items, select three of them), we have a problem with some of the possible outcomes. You see, selecting book 3, 9, and then 8 is the same as selecting book 9, 8, and then 3. It is irrelevant which book we select first or last. This situation is an example of a ***combination***.

**COMBINATION** $ \_{n}C\_{r}=\frac{ \_{n}P\_{r}}{r!}$

Items are selected in a group, meaning the order they are selected in makes no difference.

**PERMUTATION** $ \_{n}P\_{r}=\frac{n!}{\left(n-r\right)!}$

Each outcome is distinct. The order of the items within one outcome is important.

**EXAMPLE 1:** Use the correct formula to determine the correct number of outcomes for each situation above.

1. Make a three digit password B. Select three books from a list of ten

**EXAMPLE 2:** Determine if each situation is a permutation or combination. State the answer using the symbols above.

1. Choose three cereals from a selection of 15.
2. Make a 9-person batting order from 16 teammates.
3. How many different arrangements can be made if there are 9 songs on an album?
4. Select three vegetables for a dinner party from a list of 8.
5. How many different ways can twenty-five students line up for lunch?

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Permutations and Combinations

**EXAMPLE 3:** Determine the number of distinct outcomes for each event.

1. Your band has written 11 songs and plans to record 5 of them for a CD. In how many ways can you arrange the songs on the CD?
2. How many different four-person committees can be made from 18 applicants?
3. If 7 book are to be placed on a shelf, how many different ways can they be arranged?

**EXAMPLE 4:**  The movies available this month on Netflix include 9 comedies, 4 dramas, and 2 horror films. You and a friend decide to watch three movies tonight.

1. How different three-movie combinations are possible?
2. How many ways could you choose three dramas?
3. You decide to watch a horror film first, then two comedies. How many combinations are possible?
4. You decide to put the names of all the movies in a hat and draw three at random. What is the probability that you select one of each?
5. What is the probability that you select two dramas and one comedy?

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Permutations and Combinations

**PRACTICE**

1. How many ways are there to order 5 books on a shelf?
2. Simplify 10P6:
3. How many ways can we order 6 computers if we have only space for 3?
4. How many ways can we order 8 swimsuits in 4 lockers?
5. How many ways can we choose 4 t-shirts from 6 t-shirts with repetitions allowed?
6. How many 10-digit phone numbers are there?
7. How many playlists can we make with 38 songs if we can place 30 songs in each playlist?
8. How many combinations of playlists can we make with 10 songs when there are 5 songs in each and order does not matter?
9. A drawer contains 6 white t-shirts and 2 red ones. If 2 shirts are drawn at random, what is the probability of getting 2 white shirts?
10. There are 10 pink, 15 purple, and 5 green jelly beans in a jar. If two jellybeans are drawn at random (without replacement) what is the probability that both are green?

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Lesson 6.1 Predicting Outcomes

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1. 80
 | 1. 78,125
 | 1. 720
 | 1. 6,720
 | 1. 2,730
 | 1. 21,026,821,200
 |
| 1. a. 625
 |  b. 5,040 | c. 2,000 | 1. a. 10,897,296,400
 | b. 1,444,030,156,800 |  |
| c. 2,707,556,544,000 | d. 53,892,034,560 | e. 2,421,619,200 |  |

Lesson 6.2 Probabilities and Odds

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. $\frac{1}{6}$
 | 1. $\frac{1}{6}$
 | 1. $\frac{1}{2}$
 | 1. $\frac{1}{2}$
 | 1. $\frac{2}{3}$
 |
| 1. $\frac{5}{6}$
 | 1. $\frac{13}{52}=\frac{1}{4}$
 | 1. $\frac{1}{52}$
 | 1. $\frac{4}{52}=\frac{1}{13}$
 | 1. $\frac{2}{52}=\frac{1}{26}$
 |
| 1. $\frac{12}{52}=\frac{3}{13}$
 | 1. $\frac{1}{36}$
 | 1. $\frac{1}{4}$
 | 1. $\frac{1}{9}$
 | 1. $\frac{5}{18}$
 |
| 1. $\frac{1}{6}$
 | 1. $\frac{5}{6}$
 | 1. $\frac{9}{100}$
 | 1. $\frac{1}{4}$
 | 1. $\frac{3}{50}$
 |
| 1. $\frac{3}{20}$
 | 1. $\frac{1}{10}$
 | 1. $\frac{1}{25}$
 | 1. $\frac{1}{11}$
 | 1. $\frac{5}{22}$
 |
| 1. $\frac{1}{11}$
 | 1. $\frac{2}{33}$
 | 1. $\frac{1}{11}$
 | 1. $\frac{2}{11}$
 | 1. $\frac{7}{25}$
 |
| 1. $\frac{4}{25}$
 | 1. 1:1
 | 1. 11:1
 | 1. 4:3
 | 1. 4:7
 |
| 1. $\frac{6}{7}$
 | 1. $\frac{5}{11}$
 | 1. $\frac{9}{17}$
 | 1. $\frac{7}{16}$
 |  |

Lesson 6.5 Permutation and Combinations

|  |  |  |
| --- | --- | --- |
| 1. 5! = 120
 | 1. 5040
 | 1. 120
 |
| 1. 1680
 | 1. 6\*6\*6\*6 = 1296
 | 1. 10\*10\*10\*… = 1010
 |
| 1. 38P30 = 38\*37\*36\*…\*10\*9
 | 1. 252
 | 1. $\frac{15}{28}$
 |
| 1. $\frac{2}{87}$
 |  |  |